1.0

1.1

1.25

2.8

3.2

3.6

4.0

2.5

2.2

2.0

1.8

1.4   1.6

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(9) Technical memo.,

U. S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

(6) ARRAY GAIN FOR HORIZONTALLY DIRECTIONAL NOISE.

by

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USL Technical Memorandum No. 913-281-65

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INTRODUCTION

Experimental measurements were made in May 1963 (reference 1) and again for an eighteen-month period beginning in October 1963 (reference 2) of the spatial correlation function using the top twelve elements of a 40-element vertical array. The hydrophone outputs were passed through filters in the 200- to 400-, 400- to 600-, 600- to 800- and 800- to 1000 cps bands. Experimental values of spatial correlation were compared with theoretical values of spatial correlation based on the assumption directional noise sources are uniformly and independently distributed on the surface, with each noise source radiating energy as a dipole (reference 3). Agreement of the experimental results with the theoretical results is better for higher frequency bands and higher sea states. The results indicate that the noise is predominantly from the horizontal direction in the 200- to 400 cps band, while from the vertical direction in the other measured bands with the sources on the surface of the ocean radiating energy similar to a dipole source.

The purpose of this report is to describe array gain computations based on measurements of ambient noise in the low-frequency range. The first set of these measurements is spatial correlation of ambient noise. (references 1 and 2). The second set of measurements is the directionality of the noise field, (reference 4).

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ARRAY GAIN BASED ON SPATIAL CORRELATION MEASUREMENTS

Experimental Measurements

Experimental measurements of the correlation of ambient noise were made in May of 1963 (reference 1) to determine if the directional surface noise model was realistic. The resulting correlation values showed good agreement with the theoretical values in the higher frequency bands in the absence of extraneous noise sources and, in general, indicated that better agreement was obtained at higher sea states.

In the 200-400 cps band, (the lowest frequency band available), it appeared that noise from the horizontal direction might be predominating and, therefore, the model of surface-generated noise was insufficient for the low frequencies.

Beginning in October and continuing for an eighteen-month period (reference 2), measurements of spatial correlation of ambient noise were made in Bermuda to determine if the May measurements were repeatable and also if the agreement with the theoretical model was sea-state-dependent. During this period measurements were taken intermittently at various hours, no data being recorded when extraneous noise sources were known to exist in the area.

The upper 12 hydrophones of a 40-element geometrically spaced vertical array were used as receivers. The outputs of these receivers were connected to a spectrum analyzer and passed through matched Butterworth filters and then clipped. After being infinitely clipped, the filtered outputs were cross-correlated for each selected pair of receivers. The outputs were integrated and the correlation values were printed on paper tape. The clipped correlation values were then converted to unclipped values by the equation (reference 1)

$$\rho_c = \frac{2}{\pi} \sin^{-1} \rho_u \, ,$$

where ρ_c is the clipped correlation and ρ_u the unclipped correlation. The unclipped correlation values were then plotted versus the separation between the receivers divided by the geometric wavelength (Figures 1-4). (A more detailed description is given in references 1 and 2.)

COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES FOR 200-400 cps

Theoretical values of the spatial correlation function in figures 1-4 are based on setting $\psi = \tau/d_{/c} = 0$ in equation (4) of reference 5,

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resulting in

$$\rho(d,0,b) = \frac{1}{2\pi x(b-1/b)} \left\{ \frac{2\left(1 - \cos 2\pi b x\right)}{2\pi b x} - \frac{2\left(1 - \cos \frac{2\pi x}{b}\right)}{2\pi x/b} \right\}, \quad (1)$$

where

$d =$ Distance between receivers $\qquad b = \sqrt{f_1 f_2}$

$\lambda_g = \dfrac{c}{\sqrt{f_1 f_2}} =$ Geometric wavelength $\qquad x = \dfrac{d}{\lambda_g}$.

In Figures 1-4 the experimental values of the spatial correlation for different wind speeds are compared with the theoretical value in the 200-400 cps band based on equation (1). The number of samples are also noted.

By defining the array gain as

$$G = \frac{\overline{S_A^2}}{\overline{N_A^2}} \bigg/ \frac{\overline{S_e^2}}{\overline{N_e^2}}, \qquad (2)$$

where

$\overline{S_e^2}$ is the mean square output of the signal alone in one element;

$\overline{N_e^2}$ is the mean square output of the noise alone in one element;

$\overline{S_A^2}$ is the mean square output of the signal alone from the array; and

$\overline{N_A^2}$ is the mean square output of the noise alone from the array.

Becker and Cron in reference 5 have shown the array gain can be written as

$$G = \frac{1}{\sum\limits_{i=1}^{8} \sum\limits_{j=1}^{8} \alpha_i \alpha_j \rho_{ij}} \qquad (3)$$

for the eight-element array, where ρ_{ij} is the spatial correlation between the elements i and j and

$$\sum \alpha_i = 1. \qquad (4)$$

For uniform shading $\alpha_i = 1/8$ for all i and

$$G = 8^2 \bigg/ \sum\limits_{i=1}^{8} \sum\limits_{j=1}^{8} \rho_{ij}. \qquad (5)$$

In Figures 5-8 the theoretical array gain is shown for the case of an 8-element vertical array with equally spaced elements. These

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curves pertain only to vertically directional noise sources on the ocean surface with assumed dipole radiation.

The experimental points for array gain at different element spacings were computed from the experimental values of the spatial correlation coefficient available from Figures 1-4 out to $d/\lambda_s = 1.7$; interpolations were made wherever necessary. For values of ρ from $d/\lambda_s = 1.7$ to 7, the theoretical curve was used with insignificant loss of accuracy in the results, since ρ approaches zero rapidly.

It should be recalled that, in reference 5, the α_i's corresponding to the optimum shading were obtained. The gain based on the optimum shading is indistinguishably close to the gain based on uniform shading and, for this reason, all of the computations in this report were based on uniform shading.

ARRAY GAIN BASED ON DIRECTIONAL NOISE

Experimental Measurement

Axelrod, Schoomer, and Von Winkle (reference 4) employed a 40-element array in the Bermuda area to obtain the mean square of the noise as a function of frequency and direction. The essential features of the measuring equipment will be briefly described. The single elements of the array were coupled to an analog beam former. At each frequency 28 beams were formed by varying the number of elements in the array.' The outputs of these beams were coupled to a bank of 1/3-octave band-pass filters. The center frequency of these filters varied from 112 to 1414 cps. The outputs of the single elements of the array were also coupled to the same filter bank for comparison. The band-filtered beam and single element outputs were rectified, integrated, averaged and converted to a logarithmic scale. The signals were then stored on punched cards.

Simultaneously with recording the data on punched cards, an analog trace and noise spectrum were produced so that those sets of measurements containing appreciable noise from ships or whales in the vicinity of the array and from shots and active sonars could be detected. The results of this study are presented in Figure 6 (a)-(g) of reference 4 as a polar plot of noise power density with respect to frequency. In reference 4 it is pointed out that, on the basis of these figures, more noise arrives from the horizontal direction than from overhead at low frequencies, while the opposite is true at higher frequencies. Since only the horizontal direction is of interest for this analysis, only the data from the 112 cps were considered. The results for the 112 cps

4

frequency are shown in Figure 9.

ARRAY GAIN COMPUTATIONS

From the definition of array gain (equation 2) one can write the gain for the 8-element array (with the proper time delays) as

$$G = 8^2 \Big/ \frac{\overline{N_A^2}}{\overline{N_e^2}} \tag{5'}$$

Reference 4 gives the single-frequency output of a rectilinear array in a noise field as

$$\overline{N_A^2} = 2\pi \int_0^{\pi} N(\theta) \sin\theta \; g(\theta, \theta_o) \, d\theta \, , \tag{6}$$

where $N(\theta)$ is the noise intensity, $g(\theta, \theta_o)$ is the power response of the array to a plane wave with θ_o, the steering direction of the array, chosen as $\pi/2$ for this analysis.

Similarly, the single-frequency output of an element in a noise field is

$$\overline{N_e^2} = 2\pi \int_0^{\pi} N(\theta) \sin\theta \, d\theta \, . \tag{7}$$

Since the data of reference 4 apply only to the upper plane $(0 \le \theta \le 90°)$, equations 6 and 7, when substituted into equation 5, yield:

$$G = \frac{1}{\dfrac{\int_0^{\pi/2} F(\theta, \pi/2) N(\theta) \sin\theta \, d\theta}{\int_0^{\pi/2} N(\theta) \sin\theta \, d\theta}} \tag{8}$$

with

$$F(\theta, \pi/2) = g(\theta, \pi/2)\big/8^2 \, .$$

The intensity patterns for an 8-element array for various spacings are shown in Figure 10. The intensity pattern $F(\theta, \theta_o)$ was computed from an existing computer program at USL according to the equation

$$F(\theta, \theta_o) = \sum_{i=1}^{N} \alpha_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \alpha_i \, \alpha_j \, \cos\left\{2\pi x_{ij}\left[\cos\theta - \cos\theta_o\right]\right\} \tag{9}$$

where

$$x_{ij} = d_{ij}\big/\lambda \, .$$

The gain was computed by numerical methods using the values of $N(\theta)$ and $F(\theta, \pi/2)$ in one-degree increments. The gain is then

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$$G \simeq \frac{\sum_{i=1}^{90} N(\theta_i) \sin \theta_i \, \Delta \theta_i}{\sum_{i=1}^{90} F(\theta_i, \pi/2) N(\theta_i) \sin \theta_i \, \Delta \theta_i}$$

(10)

The results are summarized in Table I.

Spacing in Units of Wavelength, d/λ_g	Gain in db (Beaufort 4)	Gain in db (Beaufort 7)
1/8	0.7	1.2
1/2	3.0	4.3
1	5.6	6.2

Table I - Array Gain as a Function of Spacing in Units of Wavelengths and Beaufort Number

CONCLUSIONS

Calculations of array gain for low frequency based on spatial correlation measurements and directional noise measurements have been made.

The results derived from spatial correlation measurements are shown in Figures 5-8. The influence of sea conditions on the gain can be seen by a comparison of these Figures. With an increase in wind speed, the experimental results tend toward the theoretical values. These are expected to be sea-state dependent since they are based on the assumption that the noise is generated at the surface. For all of the element spacings of the array and all of the sea conditions considered, the gain varied from a low of 5 db to a high of 12.5 db. For comparison, a value of 9 db is obtained from the rule of thumb that $G \simeq 10 \log_{10} N$, where N is the number of array elements. The computed results are in reasonable agreement with this estimate which is only applicable to isotropic noise for element spacings equal to or greater than $\lambda_g/2$.

As a matter of general information, it has been found that, for any given geometric configuration of elements, optimum shading produces only a negligibly small increase in array gain over the method of uniform shading.

The results for array gain based on directional noise (Table I) were below those based on spatial correlation by several decibels. But the same trend with increasing wind speed was observed. It should be pointed out that the data of Axelrod, Schoomer, and Von Winkle, forming the basis of this calculation, were only available for the upper half-plane, the noise field caused by the bottom having been ignored. The present results are also valid if the noise field in the bottom is a mirror image of the field which was used in the calculations. However, for a more accurate determination of array gain by this procedure, knowledge of the actual lower half-plane noise distribution is required.

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B. Cron, Research Associate

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N. Fisch, Physicist

R. Shaffer

R. Shaffer, Research Physicist

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SPATIAL CORRELATION VS.
HYDROPHONE SPACING

FREQUENCY RANGE: 200-400 CPS

SAMPLES: 13

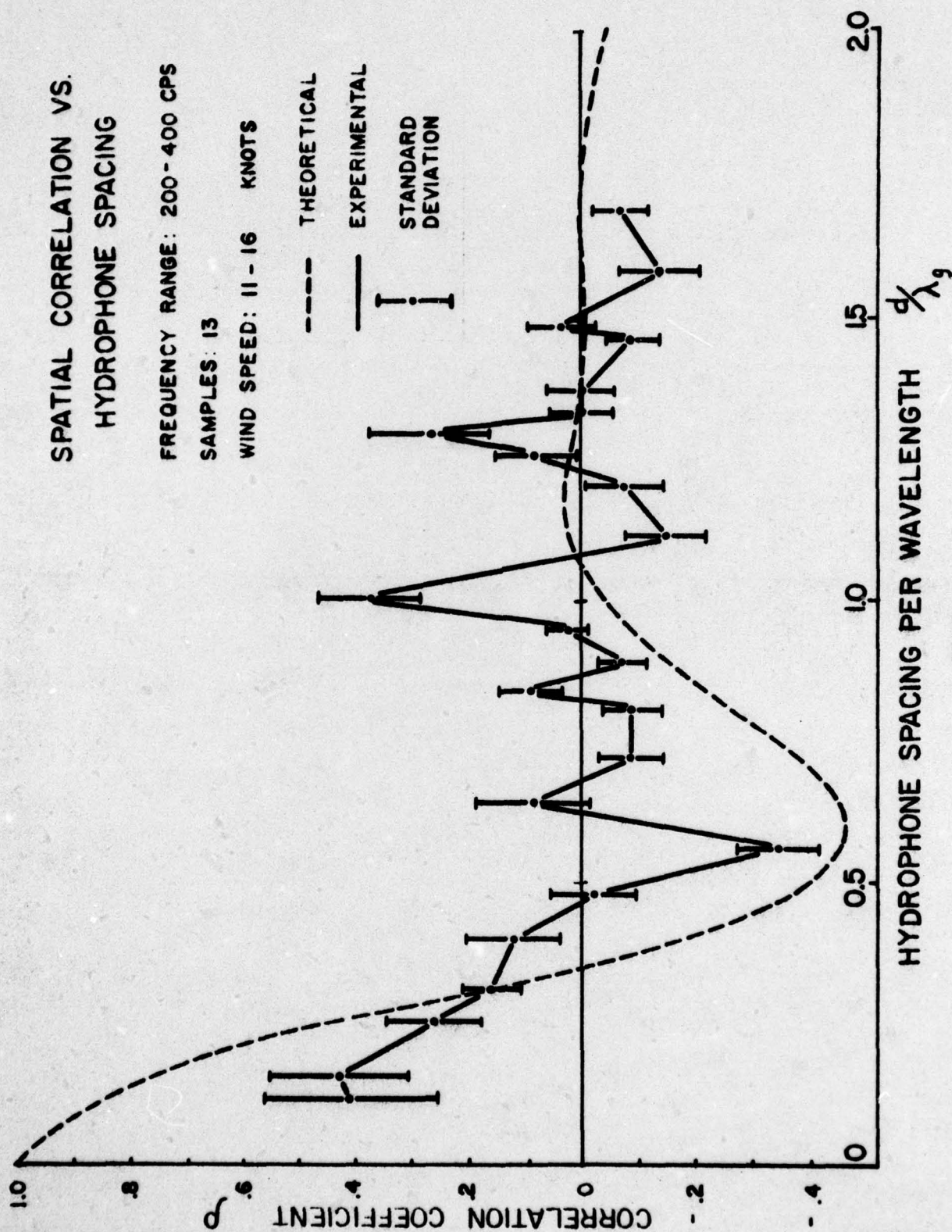WIND SPEED: 11-16 KNOTS

THEORETICAL

EXPERIMENTAL

STANDARD
DEVIATION

HYDROPHONE SPACING PER WAVELENGTH d/λ_9

CORRELATION COEFFICIENT ρ

FIG. 1

SPATIAL CORRELATION VS.
HYDROPHONE SPACING

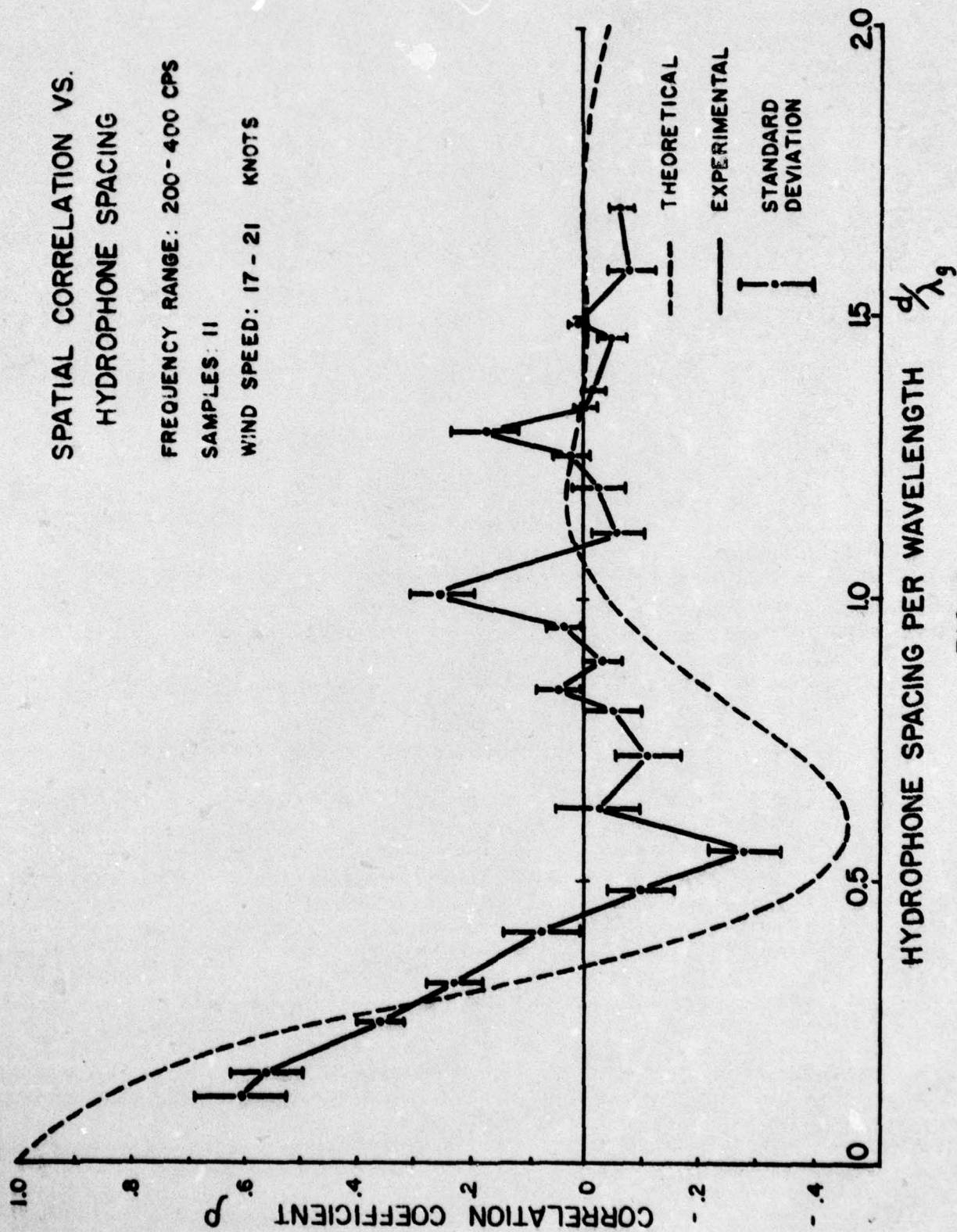FREQUENCY RANGE: 200-400 CPS

SAMPLES: 11

WIND SPEED: 17-21 KNOTS

THEORETICAL

EXPERIMENTAL

STANDARD
DEVIATION

HYDROPHONE SPACING PER WAVELENGTH d/λ_g

CORRELATION COEFFICIENT ρ

FIG. 2

SPATIAL CORRELATION VS.
HYDROPHONE SPACING

FREQUENCY RANGE: 200-400 CPS

SAMPLES: 8

WIND SPEED: 22-40 KNOTS

THEORETICAL

EXPERIMENTAL

STANDARD
DEVIATION

HYDROPHONE SPACING PER WAVELENGTH d/λ_g

CORRELATION COEFFICIENT ρ
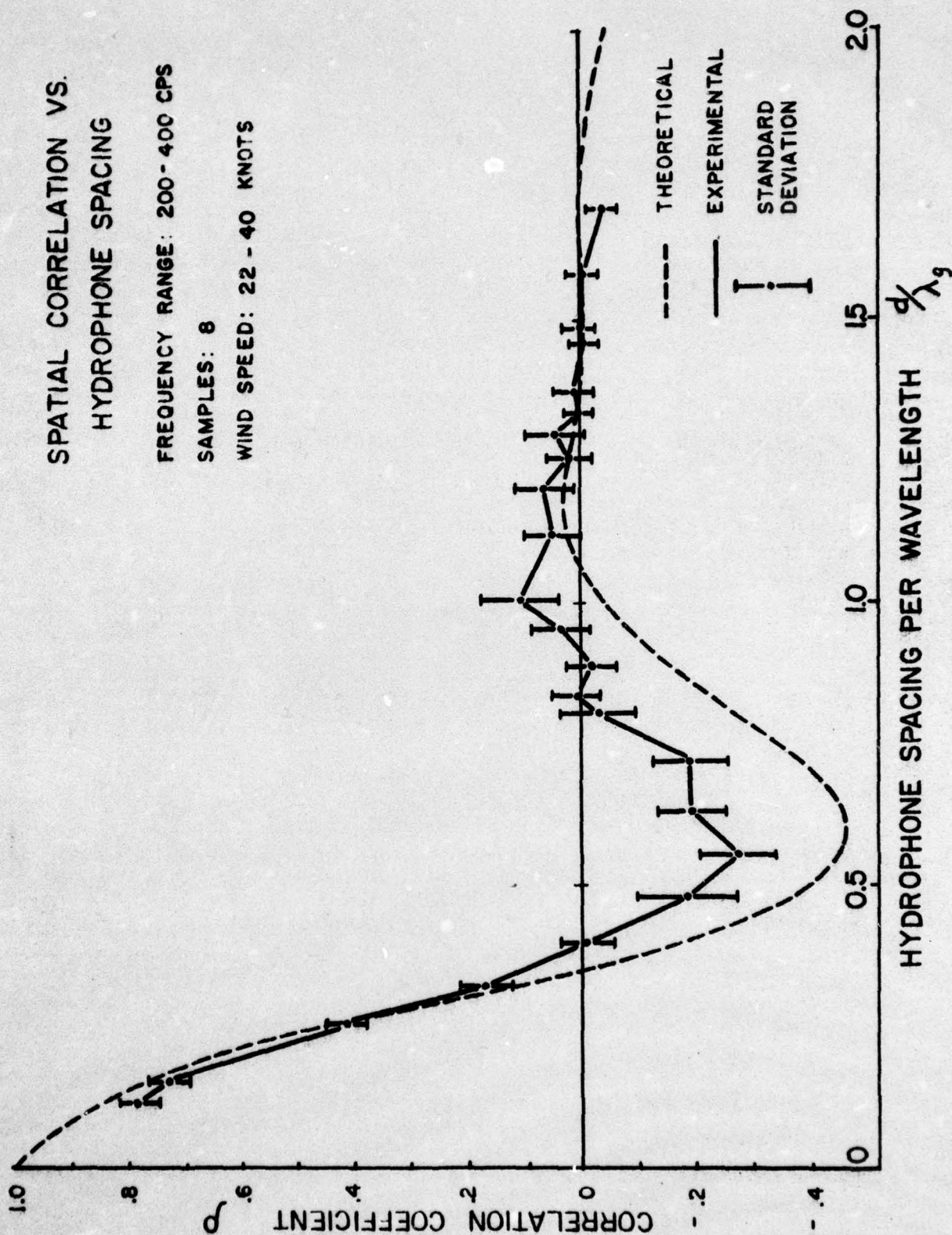
FIG. 3

SPATIAL CORRELATION VS.
HYDROPHONE SPACING

FREQUENCY RANGE: 200-400 CPS

SAMPLES: 7

WIND SPEED: 41-63 KNOTS
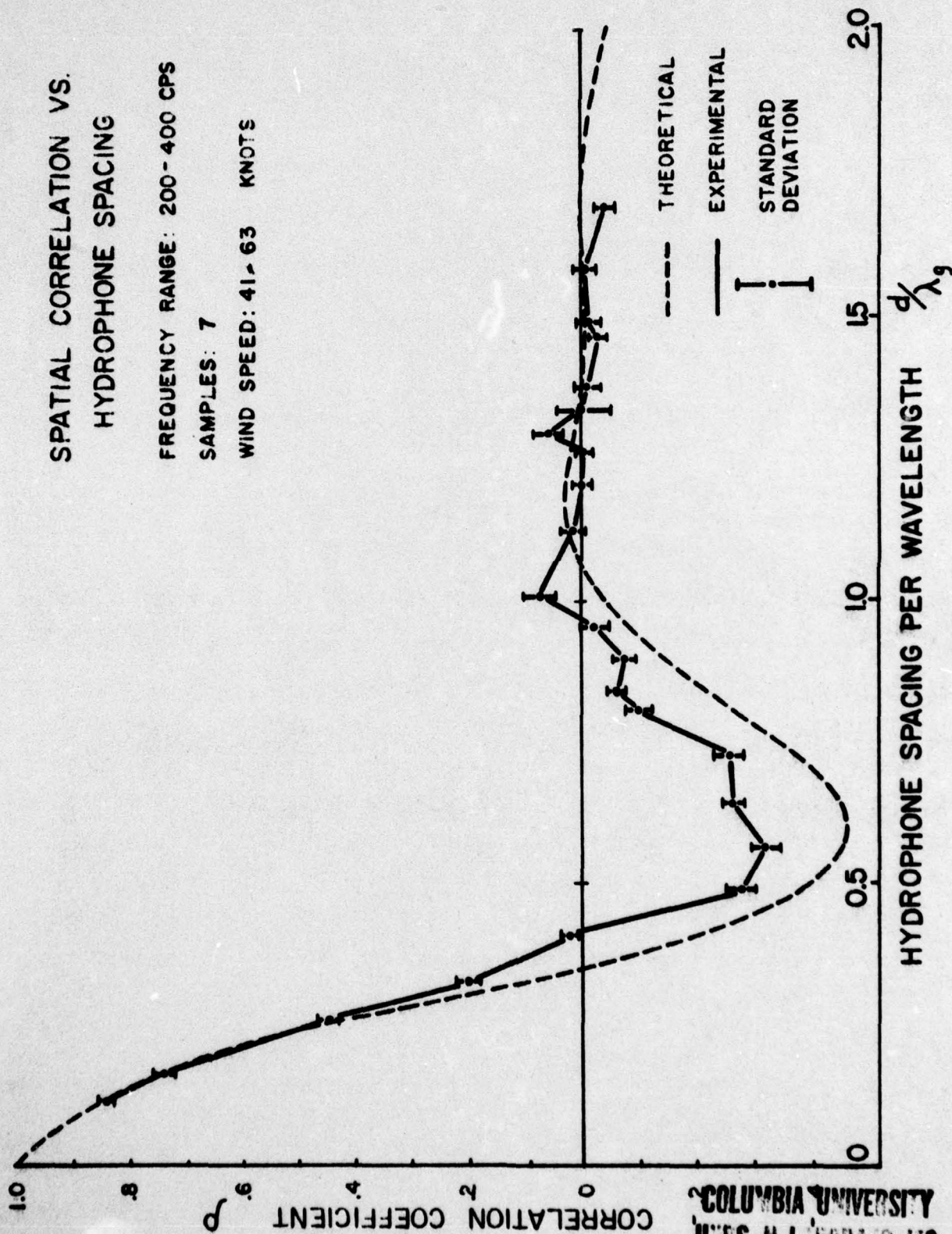
THEORETICAL

EXPERIMENTAL

STANDARD
DEVIATION

HYDROPHONE SPACING PER WAVELENGTH d/λ_9

FIG. 4

CORRELATION COEFFICIENT ρ

THEORETICAL AND EXPERIMENTAL GAIN
FOR AN 8-ELEMENT ARRAY WITH
UNIFORM SHADING

FREQUENCY RANGE: 200 - 400 CPS
WIND SPEED: 11 - 16 KNOTS
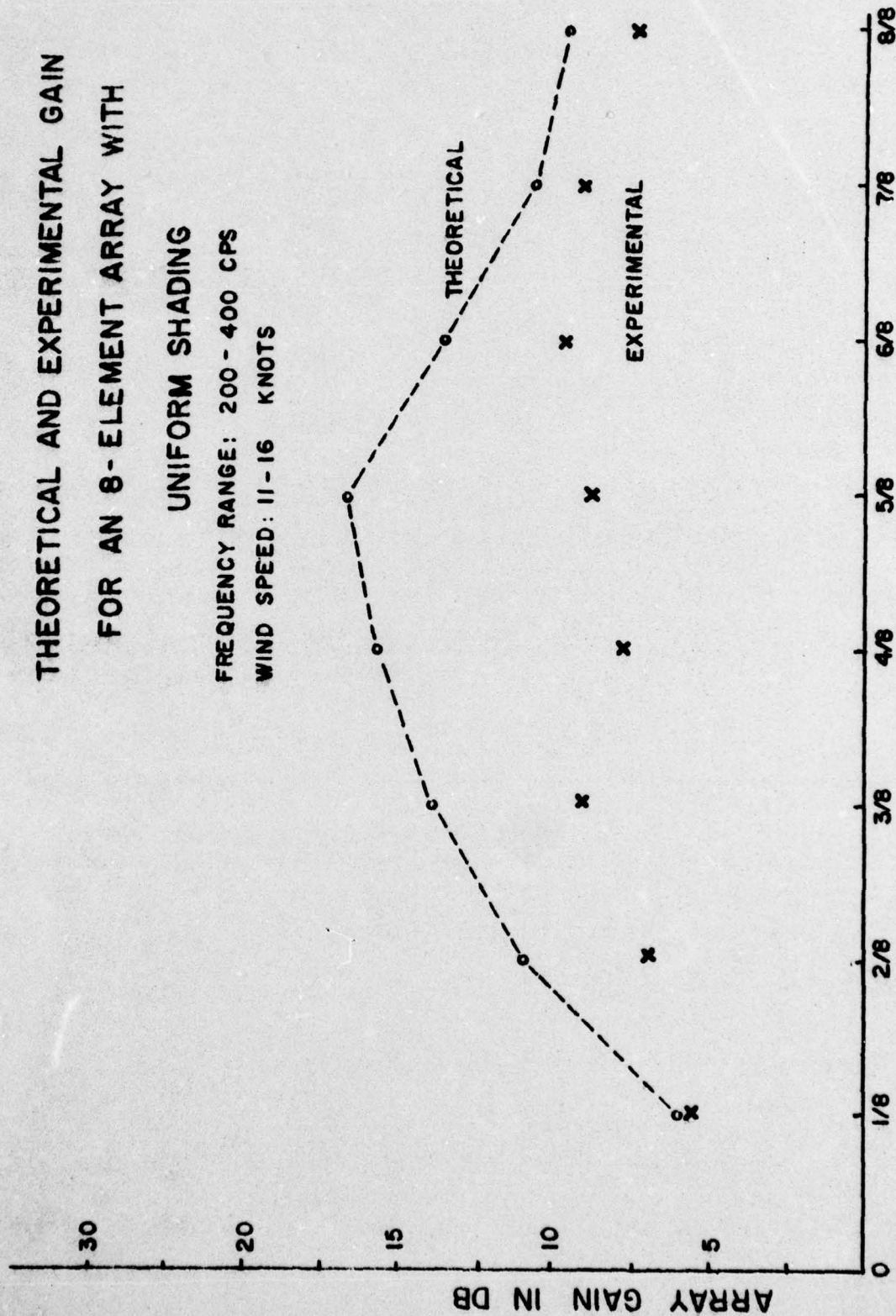
ELEMENT SPACING IN UNITS OF WAVELENGTH
FIG. 5

THEORETICAL AND EXPERIMENTAL GAIN

FOR AN 8-ELEMENT ARRAY WITH

UNIFORM SHADING

FREQUENCY RANGE: 200 - 400 CPS
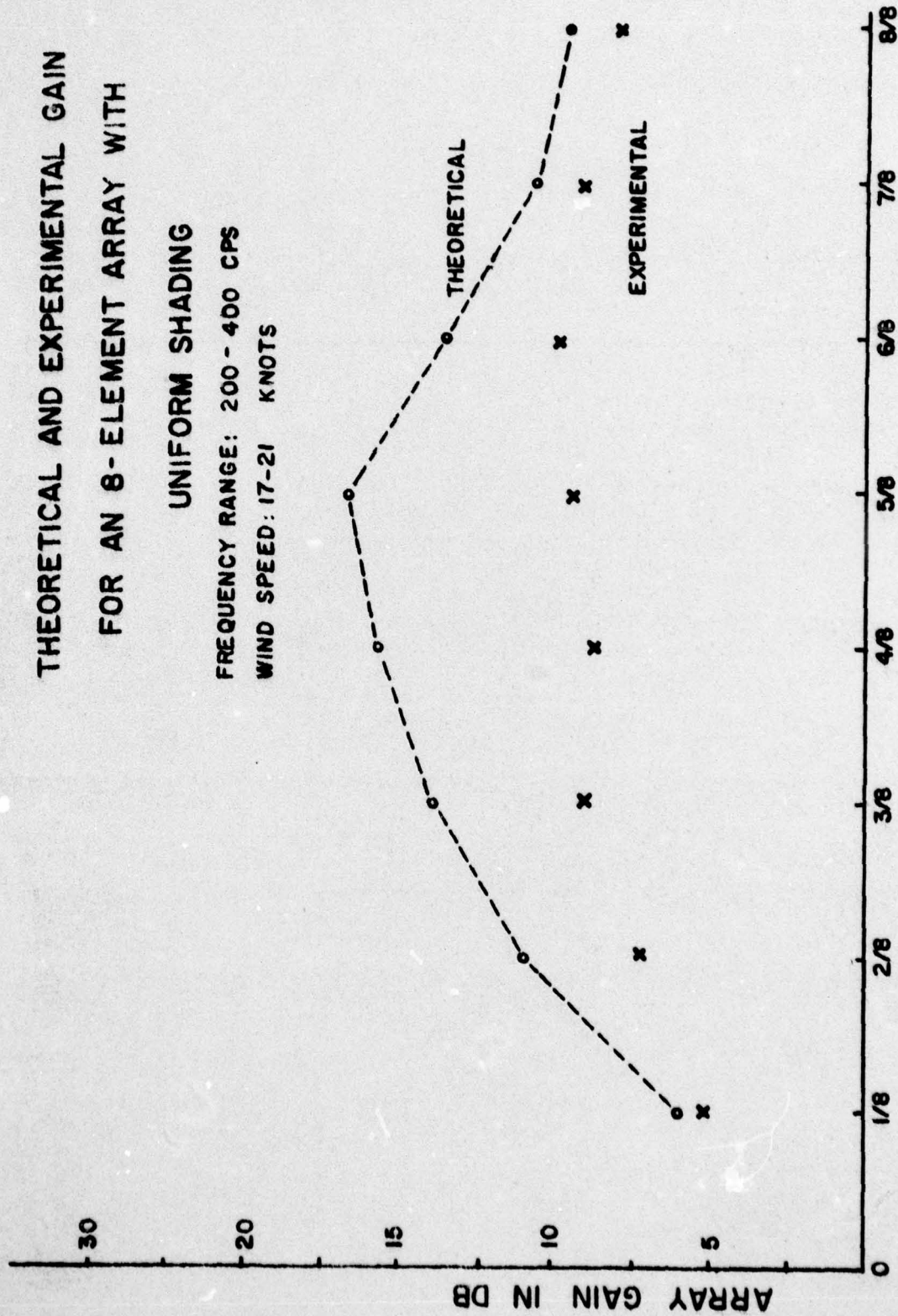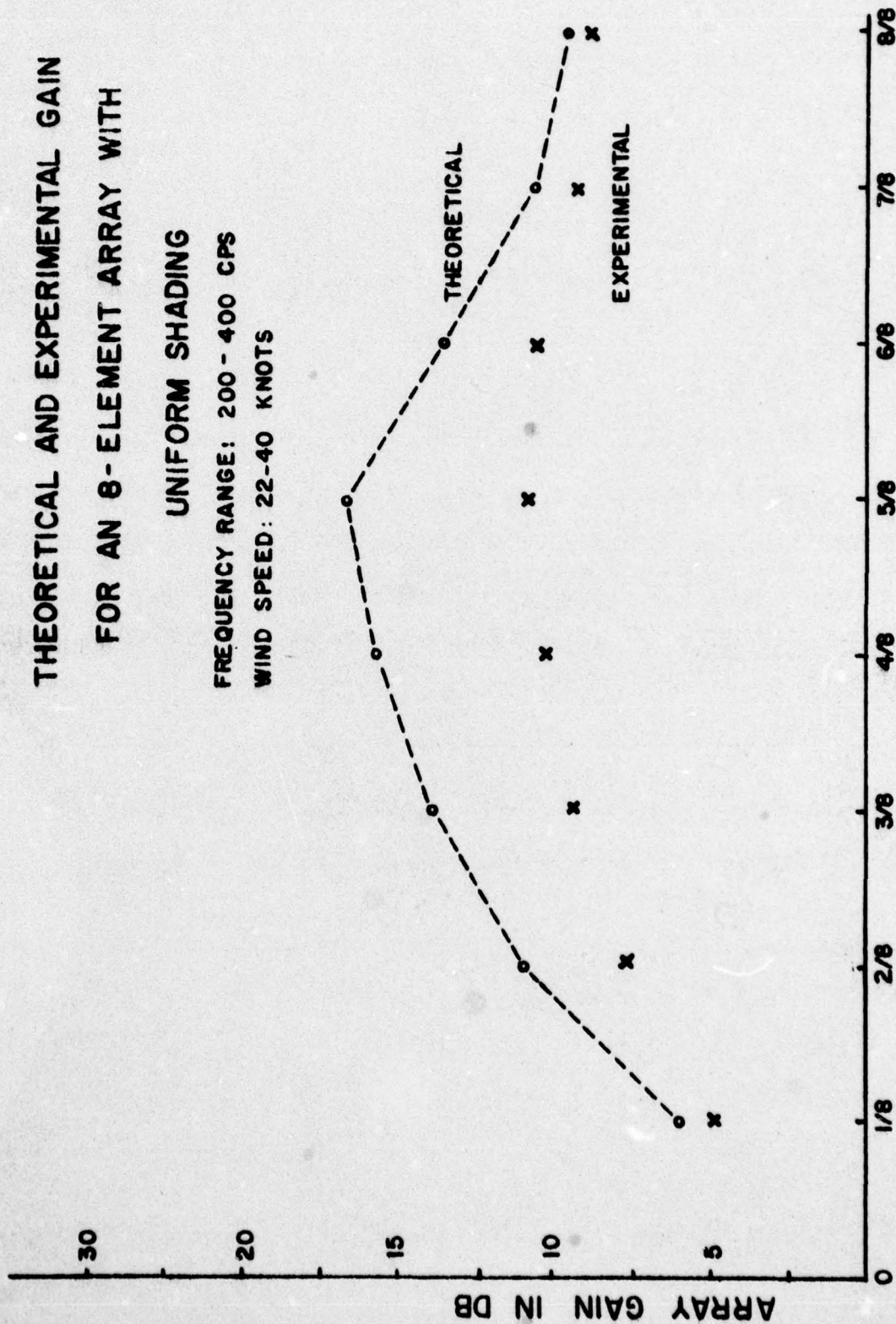
WIND SPEED: 17-21 KNOTS

THEORETICAL

EXPERIMENTAL

ELEMENT SPACING IN UNITS OF WAVELENGTH

FIG. 6

ARRAY GAIN IN DB

THEORETICAL AND EXPERIMENTAL GAIN

FOR AN 8-ELEMENT ARRAY WITH

UNIFORM SHADING

FREQUENCY RANGE: 200 - 400 CPS

WIND SPEED: 22-40 KNOTS

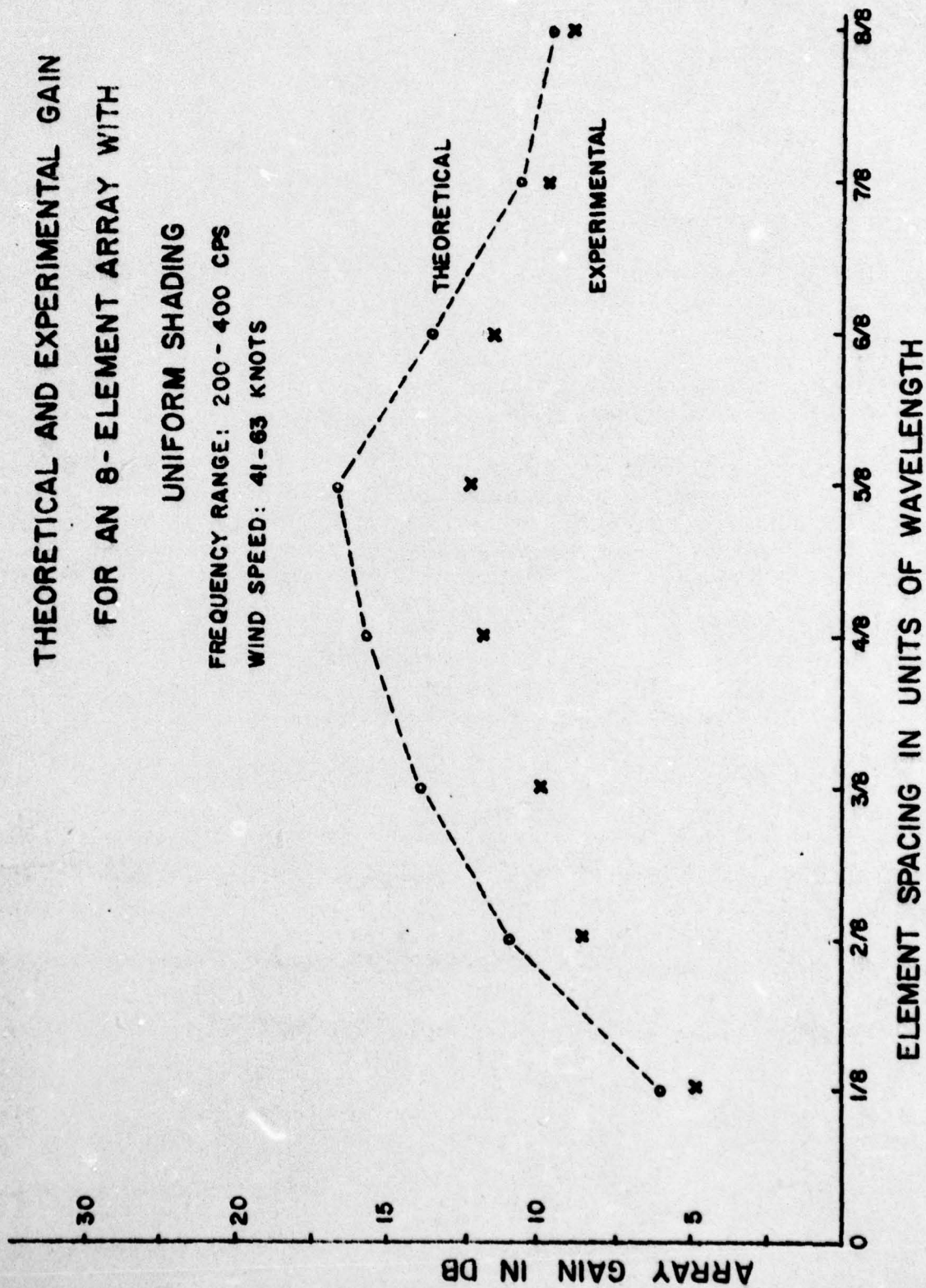THEORETICAL

EXPERIMENTAL

ELEMENT SPACING IN UNITS OF WAVELENGTH

FIG. 7

ARRAY GAIN IN DB

30

20

15

10

5

0

1/8 2/8 3/8 4/8 5/8 6/8 7/8 8/8

THEORETICAL AND EXPERIMENTAL GAIN

FOR AN 8-ELEMENT ARRAY WITH

UNIFORM SHADING

FREQUENCY RANGE: 200-400 CPS

WIND SPEED: 41-63 KNOTS

THEORETICAL

EXPERIMENTAL

ELEMENT SPACING IN UNITS OF WAVELENGTH

FIG. 8

POLAR PLOT OF NOISE, N(Θ), AT
SEVERAL BEAUFORT NUMBERS

FREQUENCY: 112 CPS

SPECTRUM LEVEL IN DB ℓ (ub)^2/ STERADIAN / CYCLE

FIG. 9

$\frac{d}{\lambda} = .125$
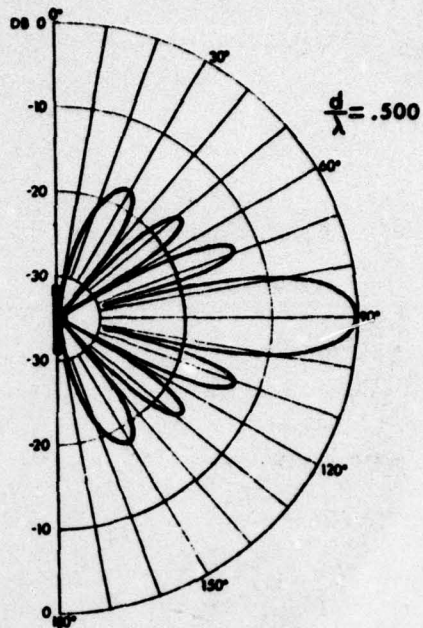
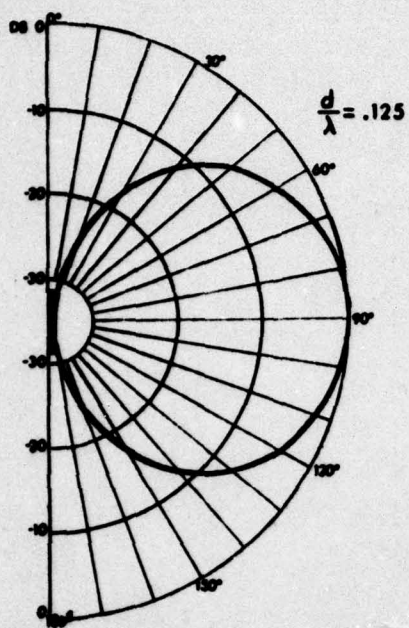$\frac{d}{\lambda} = .500$

$\frac{d}{\lambda} = 1.000$
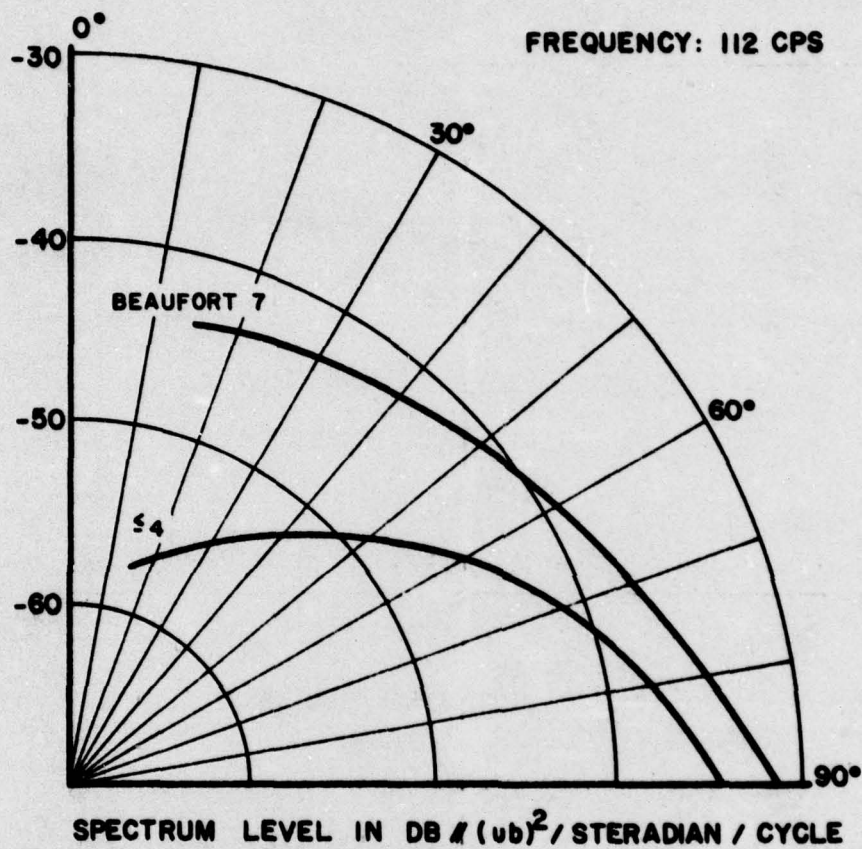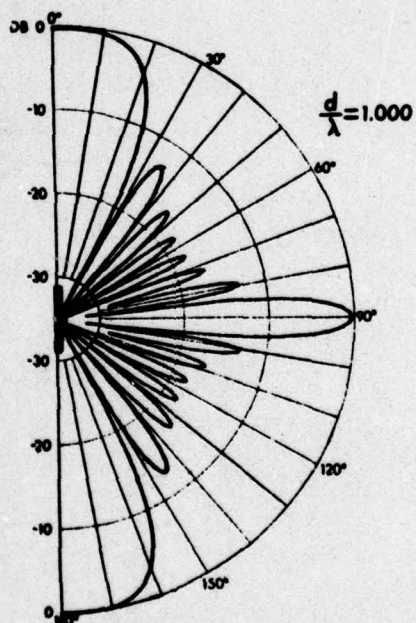
Intensity Patterns for 8 Vertical Elements and Single
Frequency with Broadside Steering

UNIFORM SHADING

FIG. 10